

Mar. 22, 2017

Sect. 6-6b

Solving Systems of Linear
Eqns. using Matrix Eqns.

Write the Matrix Eqn.

Find the Inverse Matrix
Mult.

Answer (x, y)

$$3x - 7y = 12$$

$$x + 5y = -8$$

$$[C] \cdot [V] = [A]$$

$$\begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$$\begin{array}{l}
 C \cdot V = A \\
 \cancel{C} \cdot \cancel{V} = \cancel{C} \cdot A \\
 V = C^{-1} \cdot A
 \end{array}
 \quad |$$

$$\begin{array}{l}
 C \cdot V = A \\
 C^{-1} \cdot C \cdot V = C^{-1} \cdot A \\
 I \cdot V = C^{-1} \cdot A \\
 V = C^{-1} \cdot A
 \end{array}$$

$$\begin{bmatrix} 3 & -7 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$$

$$\left| \begin{array}{cc|c} 3 & -7 & -7 \\ 1 & 5 & 15 \end{array} \right| = 15 - (-7) = 22$$

$$C^{-1} = \frac{1}{22} \begin{bmatrix} 5 & 7 \\ -1 & 3 \end{bmatrix}$$

$$\frac{1}{22} \begin{bmatrix} 5 & 7 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ -8 \end{bmatrix} \Rightarrow \frac{1}{22} \begin{bmatrix} 4 \\ -36 \end{bmatrix}$$

Note: The dimensions 2×2 and 2×1 are written in blue below the respective matrices.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -18 \end{bmatrix} \quad \left(\frac{2}{11}, \frac{-18}{11} \right)$$

$$5x + 2y = 6$$

$$6x + 4y = 4$$

$$\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$\left| \begin{array}{cc|c} 5 & 2 & 12 \\ 6 & 4 & 20 \end{array} \right| = 20 - 12 = 8$$

$$\begin{bmatrix} 5 & 2 \\ 6 & 4 \end{bmatrix} \Rightarrow \frac{1}{8} \begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix}$$
$$\frac{1}{8} \begin{bmatrix} 4 & -2 \\ -6 & 5 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 16 \\ -16 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \Rightarrow (2, -2)$$

$$2x + 4y = -2$$

$$3x - y = 18$$

$$\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 18 \end{bmatrix}$$

$$\left| \begin{array}{cc|c} 2 & 4 & 12 \\ 3 & -1 & -2 \end{array} \right| \quad -2 - 12 = -14$$

-2

$$\frac{1}{-14} \begin{bmatrix} -1 & -4 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 18 \end{bmatrix} = \frac{1}{-14} \begin{bmatrix} -70 \\ 42 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix} \Rightarrow (5, -3)$$